PATENT SPECIFICATION

DRAWINGS ATTACHED

831,066

Date of Application and filing Complete Specification Feb. 27, 1956.

No. 6063/56.

Application made in Italy on March 4, 1955.

Complete Specification Published March 23, 1960.

Index at acceptance:—Class 106(1), B5(A: F: G15).

International Classification:—G05g.

COMPLETE SPECIFICATION

A Logarithmic Computing Device

I, GIUSEPPE ARICI, an Italian Citizen, of Via Borrelli 50, Palermo, Italy, do hereby declare the invention, for which I pray that a patent may be granted to me, and the method by which it is to be performed, to be particularly described in and by the following statement:—

The present invention relates to a logarithmic computing device which is suitable for but not limited to the solution of the several inter-dependent (but not simultaneous) formulæ necessary in the calculation of the dimensions of structures made of re-inforced concrete.

According to this invention, there is provided apparatus for the solution of a plurality of equations such as those used in calculations relating to reinforced concrete structures, comprising a circular logarithmic slide rule, with additional logarithmic scales and respective co-operating indices rotatable about axes separate from the axis of the main slide rule, both the additional scales and their indices being coupled with certain respective scales of the main slide rule so as to rotate with them, and including selectively operable coupling devices for coupling each additional scale to its respective index member for rotation together.

According to a further aspect of this invention, there is provided apparatus for the solution of a plurality of equations in calculations relating to reinforced concrete and of the type in which there are more factors appearing in the equations than equations connecting such factors, the solution being based upon selecting in any way a number of factors equal to the number of equations as factors for which values are required, i.e. unknown factors; selecting a further factor as a variable factor to which a range of value is to be given and selecting for all the other factors fixed values, such apparatus comprising a plurality of circular logarithmic scales mounted on at least two axes, one scale for each factor; revolving scale supporting means for certain scales; revolving or fixed index supporting means for each scale for reading on the scale relative displacement of the scale supporting means and the index supporting means; means to set up upon the relevant scales the values assigned to each contact factor, the scales and the indices being interconnected by mechanical connecting means so that those scales which represent factors which are functions of any factor to which a value has been assigned are displaced (when the scale representing such assigned value factor is displaced) relatively to their indices and by an amount in accordance with the equations whilst those scales which are not functions of such factors are caused to remain constant with respect to their indices; and calibration means for setting up the original relative positions of certain of the scales and their scale supporting means independently of the said mechanical connections.

Many different equations can be derived to link together the various parameters which have to be taken into account in calculating reinforced concrete structures, but in order to explain the general mode of operation of the device of the present invention, four equations will be chosen as follows:—

$$d = r.\sqrt{M}.\frac{1}{\sqrt{b}} \qquad \text{A}$$

$$As = t.\sqrt{M}.\sqrt{b} \qquad \text{B}$$

$$kd = k.\ d. \qquad \text{C}$$

$$N = As\frac{1}{Av}.k^{1} \qquad \text{D}$$

Hereinafter these equations will be derived and the terms contained therein explained, but it might be mentioned that equation A appears hereinafter substantially in the form of an equation 1^1; equation B appears hereinafter substantially in the form of equation 2a; equation C is obvious and equation D appears hereinafter as equation 5^{111}.

[*Price 3s. 6d.*]

Price 4s 6d. Price 25p

For the purposes of explanation, it will be assumed that it is desired to find what are the correct values for the terms *d, As, kd, N* when values have been fixed for the terms *M, Av, r, t, k,* k^1, but when no value can be assigned to the term *b*. A consideration of the equations will show that there are four unknown factors the values of which are required, there are six constant factors to which values have been assigned and there is one variable factor which it will be assumed may have any value within a range of values.

With the apparatus of the prior invention it was necessary to select an arbitrary value for the variable factor and then calculate the unknown factors and thereafter repeat this operation for each value of the variable factor out of the possible range of values. On the other hand the present invention allows one to form partial solutions of the equations so as to obtain upon one scale the range of values of the variable factor and upon other scales (which may be adjacent or which may be linked by suitable indexes, cursors or the like) the corresponding ranges of values of the unknown factors. Thus it may be possible with a suitable choice of unknown and variable factors to obtain, upon adjacent scales, ranges of values of the unknown factors adjacent to the scale of values of the variable factor so that by merely reading from one scale to the other it is possible to obtain the solution of the equations for the range of values of the variable factor.

However, in other design calculations, it may not be possible or desirable to choose the same factors as constant or unknown as in this example and, therefore, it is a feature of the apparatus of the present invention that having regard to the four equations quoted above, any four factors may be selected as unknown factors, any other factor may be chosen as the variable factor and the remainder will be regarded as constants.

Conventionally the mechanical connections between the scale supporting means and the index supporting means comprise toothed wheels, the gear ratios of which automatically take into account the necessary variations in any factor which is a function of any other factor.

Thus, desirably the several scales form interconnected slide rules adapted to be operated by centralised controls and mounted on a single frame.

These scales are preferably not rigidly interconnected with the wheels and so do not need to be correlated with the teeth of the wheels. Thus it is not necessary to manufacture in a single unit both the scales and the toothed driving wheels. Desirably the wheels may, in certain instances, be mounted on hollow shafts which interfit one with the other and may be manufactured under the close tolerances obtainable by precision manufacture. These wheels may be provided with ground circular extensions for supporting the scales which bear the graduations and which may be made of plastic or other suitable material, and these scales may be mounted on the said ground extensions without reference to the positions of the teeth, but merely taking care that the scales are concentric with the wheels. The scales preferably do not slide one above the other but are mounted close to one another for ease of use.

Desirably when the device has been assembled calibration means (formed, for example, by friction cones arranged at important points) are freed and then locked after the scales have been arranged at accurate zero positions according to a predetermined calculation.

The precision corresponding to the zero position is reproduced in the computations since precise angles of revolutions of the wheels correspond to the spaces in the graduated scales, whilst this precision is not prejudiced by the need to manufacture the scales and wheels as unitary bodies but each can be made of suitable material and manufactured by the most suitable method.

Further features of the invention will be apparent from the following description and claims.

One preferred form of embodiment of this machine will now be described with reference to the attached drawings wherein:—

Figures 1, 2, 3 and 4 are vertical cross-sectional views of the machine;

Figure 1b is a vertical section corresponding to Figure 1 through a central pivot;

Figure 5 is a plan view of an horizontal cross-section of the apparatus taken on line V—V of Figure 1;

Figure 6 shows a plan view of the machine, showing the graduated scales; and

Figure 7 is an exploded view of the central pivot shown in Figure 1b.

With reference to the drawings, it is to be noted that the numeral references of the several parts correspond in the several figures.

This specification described both the mechanical operation and the mathematical needs solved by the several movements. For the sake of clearness, operations and determinations which occur simultaneously will be described separately.

Except for a few cases, the symbols of the "American Concrete Institute" in the "Reinforced Concrete Design Handbook", will be used herein.

1. Basic Formulæ Between the Interdependent Values of the Bending Moment (M), of the Height (d), of the Width (b), of the Reinforcement of Taut Iron (As).

The relevant theory is universally used but the formulæ deriving therefore are often written in different ways, but which can be easily transformed to one another.

The basic formulæ to be considered are:—

$$(1) \qquad d=\sqrt{\frac{M}{\frac{1}{2}\ fc.\ j.\ k.\ b}}$$

$$(2) \qquad As=\frac{M}{fs\ j.d}$$

where: fc and fs are the unit resistances of the concrete and the iron, respectively; jd is the distance of the application points of the forces$=d-\frac{kd}{3}$, k being the ratio between the height of the compressed zone (termed x in Europe and kd in America) and the height of the beam, corresponding to the expression $k=\frac{n}{n+fs/fc}$ (3), wherein n is the well known ratio between the elasticity moduli of the iron and cement.

Writing (1) as follows:—

$$d=\sqrt{\frac{2}{fc.\ j.\ k.}}\sqrt{\frac{M}{b}} \qquad (1a)$$

and calling the first term "r" where

$$r=\sqrt{\frac{2}{fc.\ j.\ k.}} \qquad (1b)$$

we have $d^2=r^2.\frac{M}{b}$, and can obtain the desired expression of (1):—

$$b=r^2.M\frac{1}{d2} \qquad (1^1)$$

Another known expression of (2) is as follows:—

$$(2a) \qquad As=t\sqrt{M.b}, \text{ wherein}$$

$$(2b) \qquad t=\frac{fc}{2.fs}\sqrt{\frac{6n}{3.fs+2\,n.fc.}}$$

and substituting therein the value b of (1^1), we will have:—

$$As=t\sqrt{M.r^2.M.\frac{1}{d2}}=t.r.\frac{M}{d} \qquad (2^1)$$

The forms (1^1) and (2^1) of the formulæ (1) and (2) are those which will be used in the following disclosure.

2. Remarks as at the Particular Needs of Solution.

It is to be noted, considering the expressions (1^1) and (2^1) that even if the values n, fc and fs (from which the terms r and t are derived) are assumed to be constant, for a given value of M there will always be two equations between the terms b, d and As; this indicates that an infinite number of solutions exists. Actually, this matter is further complicated due to variability of the resistance values fc and fs, by the insertion of a certain amount of iron in the compressed zone and so on.

The infinite number of solutions offered to the design engineer for the same problem (for instance for the same bending moment M) is a characteristic feature of computations concerning reinforced concrete and forms the advantage thereof (in that it offers the possibility of selecting the most suitable dimensions for an architectural problem) but it simultaneously forms the characteristic difficulty of this work.

In fact, at the present status of the art, although using all the tables, diagrams and special slide rules which have been provided for this purpose, the design engineer never has the possibility of examining all the solutions, completed in all of their essential parts, existing in one direction of examination.

Considering a single direction of examination (by making constant the resistance of the materials) and considering no amount of iron in the compressed zone, the design engineer must first carry out the operations, as shown at (1) or at (1^1) in order to obtain either the height d or the width b. By suitable arrangements of slide rules, or by special tables or diagrams, he can also examine all the infinite combinations of d—b of the two elements height-width, without however simultaneously examining the other determinant element (which is formed by the iron As) relative to the several combinations of d—b. In order to know at least the three determinant elements d—b—As, he will have to examine first d—b and select then without knowing As. He then has to operate on the expression (2) by the selected values of d—b in order to find As. Now, as the selection of a proper solution depends upon the simultaneous knowledge of d—b—As, there follows that, at the present status of the art, the operator is hindered in the function which is required of him. This function could, on the contrary, be fully carried out if the operator could see the infinite range of the possible solutions which are composed by all of the elements d, b, As, and, better still, if he could read also the distance kd and either the area or the number of the vertical reinforcements.

3. Specification and Operation for Solving the Two Basic Formulæ

Assuming that a knob 1 (see Figures 2 and 6) is slightly rotated without depressing it, the shaft of the knob rotates and carries with it a small toothed wheel 2 (see Figure 2) which rotates a large toothed wheel 3. As a spring 4 has not been compressed, it presses a male cone 5 on the shaft against the inside of a female cone 6 to cause rotation of the cone 6 which carries teeth adapted to rotate a large toothed wheel 7.

The tooth diameters of the wheel 2 and cone 6 are equal to one another and the diameters of the wheels 3 and 7 are equal, so that the wheels 3 and 7 rotate through the same extent. The wheel 3 causes rotation of a hollow shaft 8 mounted on a stationary shaft 9 secured to the base 24 of the device. The hollow shaft 8, by means of a friction cone 10, is normally rigid with a disc 11 carrying a transparent cursor *ir* provided with a hair line. In turn, the wheel 7 is rigid with a hollow shaft 12 loosely mounted on the shaft 8 and carrying a graduated scale *r*.

As aforesaid, the wheels 3 and 7 rotate together and therefore the scale *r* and the transparent cursor *ir* will also rotate together, the cursor therefore showing a constant value on the scale *r*.

The wheel 7 (see Figure 1) rotates an identical toothed wheel 13 rigid with a spider bearing 72 which carries a graduated scale *M* fixed thereon. Simultaneously, the wheel 3 drives an identical toothed wheel 14 which is rigidly connected to a bearing 73 screwed at 16 to a carrier 17, for a scale *d*. Under these conditions, scales *M* and *d* turn together by the same amount. The wheel 13 (see Figure 4) also engages a smaller wheel 15 rigid with a female cone 18 which, by the action of a spring 19, drives both a male cone 20 and its toothed wheel 21. The wheel 21 engages a larger toothed wheel 22 which revolves in synchronism with the wheel 14.

The wheel 22 is rigidly mounted on a shaft 23 which is secured at its upper end, by a nut 74, to a transparent cursor *ip* provided with a hair line (see Figure 6). Thus, under these conditions the cursor *ip* rotates exactly with the graduated scale *d* and therefore its hair line constantly shows the same value on *d*. The frame 24 carries, by suitable projections, a bearing 25 (see Figure 1) which supports the hollow shafts which rotate inside and outside it. The bearing 25 is rigidly secured by a screw thread 26, to a box 27 the inside of which carries an elastic ring 28 (see Figures 1 and 1*a*) which can be controlled as it will be explained herebelow so as to be caused to engage with an inner ring 29 (which is rigidly connected to the cursor *ip* by means of connections 75) while disengaging the box 27 or, alternatively, to discharge the ring 29 (and the cursor *ip*) and remain with its outer edge fixedly engaged with the fixed box 27.

A rivet 79 secures elastic ring 28 to a ring 30 carrying a graduated scale *b*. Thus when the ring 28 is stationary and the graduated scale *b* is also stationary the cursor *ip* lies over the scale *b* and, aforesaid, due to the drive through the wheels 14, 15, 21 and 22 (see Figure 4) revolves to the same extent as the scale *d* and the scale *M*, so that the cursor *ip* shows on the scale *b* an angular distance equal to that shown on *M* by a suitable cursor *iM* (see Figure 4) fixed to the frame.

In order to examine the effects of the aforesaid movements, the clockwise movement will be termed "right-hand movement" while the counterclockwise movement will be termed "left-hand movement".

As shown in Figure 6, the scale *b* is a left-hand scale and is a double scale, i.e. has two complete logarithmic scales on its circumference. Also the right-handed scale *M* is double.

If the knob 1 is rotated so as to impart a certain left-handed revolution to the scale *M*, an increasing value will be read under the line *iM*. Assuming that the rotation is made from 1 to 2, simultaneously the cursor *ip* will rotate through an equal amount to the left, and said amount will be read on *b*, as the scale *b* is also double but is a left-hand scale. If the line of the cursor *ir* is on the digit 1 of the scale *r*, and if the line of the cursor *ip* is on the digit 1 of the scale *d*, the solution will be obtained:—

$$b=M\times 1=1(1^{II}).$$

This is the correct solution of equation (1^{I}) when $r=1$ and $d=1$.

If the scale *M* is turned back by the knob 1 so as to have the value 1 under the line *iM*, simultaneously the hair line of cursor *ip* will be carried back to the value 1 on the scale *b*.

If the knob 1 is again actuated, but in a left-hand direction and whilst depressed so as to compress the spring 4, the female cone 6, the wheel 7 and the graduated scale *r* will remain stationary. On the other hand, the wheel 2 will turn the wheel 3 to the right causing the right-hand movement of the cursor *ir* over the scale *r*. As will be disclosed hereinafter, both the scale *r* and the cursor *ir*, as shown in Figure 6, have a particular composition. Let us assume, temporarily, in order to understand the operation that on *r* there is a common right-hand logarithmic scale extending throughout a complete circumference, instead of half a circumference as on the scales of *b* and *M*.

With the knob 1 depressed, if the cursor *ir* is caused to move from 1 to 2 on the scale *r*, the wheel 3 will equally rotate (through an equal but left-handed extent) both the wheel 14 and the scale *d*, and through the drive chain 14, 15, 21 and 22 also the cursor *ip*.

The wheel 7 will be stationary and so will both the wheel 13 and the scale *M*. Then, on *M* will be read 1; on *r* will be read 2; *d* and *ip* have rotated together and on *d* will be read always 1; on *b* will be read $4=2^2$ as *ip* has rotated to the left, on the left-hand scale *b*, extrusion of which is double. If, on the con- through an amount 2 read on scale *r* the trary the knob 1 is rotated so as to read 4 on *r*, on *b* will be read 16 i.e. 4^2, and the solution of equation (1^{I}) will be embodied as follows:—

$$b=r^2.M\frac{1}{d^2}=r^2\times 1\times 1=r^2 \qquad (1^{III}).$$

Now, releasing the knob 1, a knob 47 (see Figures 4 and 6) may be pressed and turned;

by compressing the spring 19, both the female cone 18 and the wheel 14 remain stationary, and the knob 47 only rotates the wheel 21 and the wheel 22, this latter rotating the cursor *ip* and its hair line simultaneously over the scales *d* and *b*. As shown in Figure 6, the scale *d* is a right-hand scale (contrary to *b*) and is a single scale, while *b* is a double scale, so that when the rotation of the knob 47 has caused *d* to pass from 1 to 2, the value shown on *b* by *ip* will be divided by 2^2, so that if said value was 16 (when *r* was on 4) it will be $4=\frac{16}{2^2}$ when *d* will reach the value 2.

Equation (1^{I}) will be solved so as to have at *b* a value:—

$$b=r^2\times 1\times\frac{1}{d^2} \qquad (1^{IIII})$$

This is the solution of equation (1^{I}) if, as in this case, $M=1$.

On releasing the knob 47, we have:—

on *M* one reads the value 1

on *r* one reads a certain desired value (assumed 4)

on *d* one reads a certain desired value (assumed 2)

on b one reads a value $b=r^2\times 1\times\frac{1}{d^2}$

We can now summarize the solution of first formula; according to the matter above disclosed, if the knob 1 is revolved without pressing, there will result the movement of *M* and, as aforesaid, all of the angular segments that it shows will be logarithmically added to the value shown by *ip* on *b* according to equation (1^{IIII}) so that, for any value shown by *iM* on *M*, on *b* we will have the value according to equation (1^{I}) $b=r^2 . M\frac{1}{d^2}$.

This last cited movement, serving to cause the scale *M* to pass from the value 1 to any value, while *b* passes from $r^2\frac{1}{d^2}$ to $r^2.M\frac{1}{d^2}$ to solve the formula (1^{I}) simultaneously causes other movements which also solve the formula (2^{I}). Particularly the left-handed revolution of the wheel 13 (for positive increments of *M*) turns an idler pinion 31, (see Figure 5) to the right. Said pinion 31, which is idle on its shaft, has the sole function of imparting a left-handed revolution to a wheel 32 owing to the left-handed revolution of the wheel 13, the aforesaid wheels being equal to one another. By means of an arrangement similar to that of Figure 2, shown in Figure 3, the wheel 32 rotates a bearing 33 carrying a graduated scale *t*.

The teeth of the wheel 32 mesh with the teeth of a female cone 34 which, under the action of a spring 35, causes rotation of a small toothed wheel 36 which rotates a large toothed wheel 37 through an equal extent and in the same direction as the wheel 32.

The wheel 37 is rigidly mounted on a hollow shaft 38 which turns about a pivot 39, the wheel 37 being connected by a friction cone 40 to a disc 41 carrying a transparent cursor *it* having a hair line. As the wheels 32 and 37 revolve together, the cursor *it* shows, on the scale *t*, a constant value which may be assumed, now, to correspond to 1.

The wheel 37 engages an equal wheel 42, (see Figure 1) said wheel 42 being rigid with a bearing 43 which is rigidly secured by a screw thread 44 to a disc 45 carrying a graduated scale *As*, the values of which are shown by an extension of the cursor *ip* showing the values *d* and *b*.

Assuming that in its starting condition, when *M*, *r*, d, and *b* were all on 1, *As* also was on 1, the displacements of *As* and the values which can be read thereon are as follows:—

As (see Figure 6) has a single left-handed logarithmic scale extending throughout its circumference. When the knob 1 was pressed and rotated, *M* remained on the value 1 and both the wheel 13 and the scale *As* remained stationary. However, the wheel 3 was rotated through a right-hand amount *r* shown on a simple scale, while the wheel 14 with the scale *d*, as well as the wheel 22 with the cursor *ip* was rotated through a corresponding left-hand amount; after such a movement, the cursor *ip* on the simple left-hand scale *As* showed a value *r* and we may read:—

$$As=r \qquad (2^{II}).$$

If, leaving *M* (and therefore *As*) stationary, the knob 47 is actuated and pressed so as to move the cursor *ip* with respect to the stationary scale *d*, so as to read any value, the cursor *ip* in order to show said value instead of 1, moves a right-hand angular displacement on the right-handed simple scale of *d*. This operation subtracts the logarithmic segment *d* from the value marked for *As*. Thus, after this operation, on *As* will be read not only the value *r* of equation (2^{II}) but also the value:—

$$As=r\,\frac{1}{d} \qquad (2^{III}).$$

Lastly, when the knob 1 is rotated, without pressing, in order to move the scale *M* from the value 1 to any value, the cursor *ip* is also moved to the left through an amount *M*, shown on a double scale, so that to the value *As* of equation (2^{III}) is added a logarithmic segment $\frac{1}{2}$ M.

As simultaneously the wheel 13 is moved, by the aforesaid connections (13, 31, 32 . . .), the left-handed scale *As* is rotated through an equal amount and in the right-handed direction contrary to the cursor *ip*, and therefore the value of the scale has been increased of an amount corresponding to $\frac{1}{2}$ M, so that by the movement of M, the value *As* has been incremented by 2.½ M, i.e. by M. Therefore, the value read on *As*, after this operation is:—

$$As=r\frac{1}{d}M \qquad (2^{1111}).$$

By comparing the expression (2^{1111}) with the expression (2^1), we note that only the factor *t* is missing. In fact it was assumed that the pointer *it* was stationary on the value 1 of the scale *t*. If a knob 46 is now rotated under pressure in order to compress the spring 35, the female cone 34 remains stationary as does the graduated scale *t* and all the graduated scales *M*, *r*, *d*, and *b*, while only the wheel 36 rotates, together with the cursor *it*.

The graduated scale *t* also has particular features, as has the scale *r*, and will be described hereinafter. For the sake of simplicity it will be assumed that the scale *t* is formed as common single left-handed logarithmic scale. If the knob 46 is rotated in the right-handed direction, under pressure, in order to rotate the wheel 37 in the left-hand direction through a certain amount *t*, shown by the cursor *it* on the scale *t*, both the wheel 42 and the left-hand scale *As* rotate through an equal right-hand amount.

The pointer *ip* is stationary but the right-handed movement of *As* corresponds to a left-handed relative movement of the cursor *ip* on the left-handed scale of *As*; wherefore the angular logarithmic value *t* is added. Finally, therefore, on *As*, we may read:—

$$As=r.t\,M\frac{1}{d} \qquad (2^1)$$

The result of the above is that when:—
ir indicates any value *r*
it indicates any value *t*
ip indicates any value *d*
a simple rotation without pressure, of the knob 1 moves under *iM* any value of *M* and, at the same time, moves under *ip*, the values *b* and *As* according to the equations (1^1) and (2^1).

The operator in order to know the first solution, complete with *d*, *b*, *As* and, as will be explained hereinafter, also with *kd* and the number of stirrups, need only rotate the knob 1 until he reads under *iM* the value of *M* of his problem.

In order to scan all the infinite solutions of this problem *M*, without varying the resistance of the materials (*r* and *t*) the operator leaves the scales stationary on this first position. It is sufficient to operate by the knob 47, pressing and rotating this knob when all the scales and the toothed wheels (except the wheels 21 and 22) remain stationary and the wheel 21 causes the wheel 22 to rotate, rotating thereby the cursor *ip*.

The variations carried out by this revolution on the values of the indications given by the cursor *ip* on *d*, *b*, and *As* (and simultaneously on *kd* and on the number of stirrups, as it will be explained hereinafter) will be discussed herebelow.

The values shown on *M*, *r*, and *t* do not change. If it is assumed that the cursor *ip* is moved to the right through a certain logarithmic angle, by pressing and rotating the knob 47, said operation being, for instance, that necessary for passing from the original position *d*=2 to *d*=4, this logarithmic amount 2 which added to *d* has multiplied its value by 2 on the scale *d*, will be doubly subtracted as the scale *b* is a left-hand scale and as the logarithmic base of the scale *d* is 360° and is therefore double the logarithmic base of the scale *b* which is 180°. In other words, the pre-existing value of *b*, which was assumed to be 4, is divided by 2^2 and becomes 1. Thus, during this operation wherein *r* and *M* remain constant, any position reached by cursor *ip* satisfies the equation:—

$$b=r^2.M\frac{1}{d^2} \qquad (1^1)$$

The scale *As* is also a left-hand scale, contrary to *d*, but having an equal logarithmic base (360°) and therefore for any logarithmic angular segment added to the values of *d* by the displacement of cursor *ip*, said segment will be equally divided into the values of *As*. In other words, while *r*, *t* and *M* are constant, all positions of cursor *ip* will satisfy equation (1^1) and also the equation:—

$$As=r.t.M\frac{1}{d} \qquad (2^1)$$

It is therefore possible to scan, by a free revolution of the cursor *ip* all of the infinite solutions of the problem, these solutions being complete with the factors *d*, *b* and *As* which, when fully known, permit the operator to choose readily and well.

As aforesaid, beside the factors *d*, *b* and *As*, the operator reads also the distance *kd* and either the area of the number of the vertical reinforcements *Av*. How this result is obtained will be described herebelow.

4. Simultaneous Reading of the Spacing of the Neutral Axis *kd*.

The carrier 17 also carries, besides the scale *d*, a graduated ring *s*. The carrier 17 also carries a knurled knob 48, whereby it is possible to rotate a toothed pinion 45 meshing with a toothed annular ring 50, carrying a graduated scale *kd* and moving within a suitable channel in the carrier 17 to which it is secured by springs 51 held by screws 52.

The graduated ring *s* is provided with two central scales *k* (see Figure 6) and with two outer scales, with smaller digits, of which outer scales a scale F^1 is the outer one, while a scale *Cr* is the inner one.

At the beginning of the present specification, I have shown the value *k*:—

$$k=\frac{n}{n+\frac{fs}{fc}} \qquad (3)$$

and said value is therefore a quantity depending upon the values n, fs and fc, concerning the resistance of the materials.

The characteristic feature of these scales will be described herebelow, as well as for the scales r and t. For sake of clarity it is assumed that on k there is a common left-hand single logarithmic scale (base 360°) of the values k. The values of this scale are indicated by a transparent cursor *is* carrying a hair line and rigidly secured to both the toothed ring 50 and its scale *kd*.

If one imagines a starting position wherein the cursor *ip* shows the value 1 on the scale *kd* and simultaneously the value 1 on the scale d while the cursor *is* shows the value 1 on the scale k of the ring s, if the knob 48 is actuated so as to turn both the ring 50 and the cursor *is* to the left when cursor *is* is marking a value k on the scale k, the scale *kd* will be moved through an equal extent with respect to d. In other words, while cursor *ip* still marks 1 on scale d, it will mark k instead of 1 on scale *kd*. When cursor *ip* instead of marking 1 marks on the scale d any given value of d, it will mark on the scale *kd* the corresponding value *kd* instead of k.

5. VERTICAL REINFORCEMENTS *Av* (STIRRUPS).

The diagram of the shearing stresses on an uniformly loaded beam has the shape of two right angled triangles, each of which has a length equal to one half of the length of the beam. The shearing stress having zero value at the centre of the beam and a maximum value V^1 at its ends. The present formulæ for determining the number of stirrups and of the distance s for a determined stirrup having the area *Av*, are always obtained by the fundamental formula:—

$$Av=\frac{V^1\ s}{fv.j.d} \qquad (4)$$

where s is the length of the portion of the beam concerning the area *Av*; *fv* the resistance to shearing stress of the iron which is often considered to be $^4/_5$ of the resistance to tensile stresses; and j and d have the usual meaning (see Number 1).

The number N of stirrups, having a cross-sectional area *Av*, which are necessary in one half of the beam for taking the shearing stresses on a whole triangular diagram is:—

$$N=\frac{1}{2}\cdot\frac{L}{2}\cdot\frac{v^1.b}{Av.fv}$$

and, for the complete beam (two triangular diagrams):—

$$N=\frac{L}{2}\cdot\frac{v^1.b}{Av.fv.} \qquad (5)$$

where L is the length of the beam and

$$v^1=\frac{V^1}{b.j.d}=\text{maximum unit stress} \qquad (6)$$

Now, the possibility of avoiding for the operator the necessity of solving this formula (or other equivalent formulæ) has been found and applied to this device, in order to know the number N of necessary stirrups (or the total cross-sectional area $N.Av$) when the elements M and As are already known, on condition that an always known element be kept present and precisely the ratio $\frac{P.L}{M}$ (which will be termed "binding condition", where P is the total of the uniform weight supported by the beam.

The most common binding condition is 12. This value will be considered, by way of example. The equation (6) can be written as follows:—

$$v^1.b=\frac{V^1}{j.d} \qquad (6^1)$$

and, by substituting the values in equation (5) one obtains:—

$$N=\frac{L}{2}\cdot\frac{V^1}{j.d}\cdot\frac{1}{Av.4/5.fs} \qquad (5^1)$$

considering $fv=4/5.fs$.

It is known that $V^1=\frac{1}{2}$ P and, owing to the established binding condition $M=\frac{P.L}{12}$; by substituting $M=\frac{1}{6}\ V^1\ L$, or $3\ M=\frac{V^1.L}{2}$ in equation (5^1), one obtains:—

$$N=3\frac{M}{j.d}\cdot\frac{1}{Av.4/5.fs} \qquad (5^{11})$$

and, as $As=\frac{M}{j.d.fs}$ from equation (2), the equation (5^{11}) is transformed to:—

$$N=\frac{3}{4/5}\cdot\frac{As}{Av} \qquad (5^{111})$$

One may demonstrate that this equation is true even if the load is concentrated either in the central zone or at any point, and therefore said equation has been utilized in the apparatus according to this invention, as said equation indicates that when the cursor *ip* indicates on the scale *As* any value of the taught reinforcement necessary for a beam (for any moment M) if this moment was calculated under the binding condition $M=\frac{P.L}{12}$ for a subdivided load or generally $\frac{2}{3}$ of the load due to the simple rest, then the number of the stirrups will be simultaneously indicated, on the same scale As, by another transparent cursor iN, carrying hair line, rotated with respect to the cursor ip through an amount:—

$$\frac{3}{4/5} \cdot \frac{1}{Av}.$$

If it is assumed that stirrups have a cross-sectional area $Av=1$, it will be sufficient that the line iN be rotated through a constant amount: $\frac{3\times5}{4}$.

In the countries using the decimal metric system, the stirrups will be 4, 5, 6, 7, 8 or 10 mm. diameter, and in the countries using the British system, the stirrups will be $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$ inches diameter, or $\frac{1}{2}$ sq. in. Therefore, on the cursor ip, wherein the cursor iN rotates, are marked by the digits 4, 5, 6 and so on, the logarithmic angular segments corresponding to the cross-sectional area As of a two branched stirrup, said branches having a diameter of 4, 5, 6 and so on mm. all subtracted by a constant displacement equal to: $15/4=3{,}75$.

As these angular segments are taken in the right-hand direction, opposite to the left-hand direction of As, when the extension of the hair line of cursor iN is on 4, 5, 6 and so on, the same line will indicate on the scale As a number which will be exactly the one according to equation (5^{111}):—

$$N=As.\frac{15}{4} \cdot \frac{1}{Av}$$

Analogously, for the countries using the British system, one may immediately read the number of stirrups of $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$ or $\frac{5}{8}$ inch and so on.

This possibility is extended to all the other binding conditions (other than the common case of a condition equal to $\frac{2}{3}$) and to all the values of the ratio $\frac{fv}{fs}$ (other than the common ratio 4/5) by the addition of a small logarithmic scale a rigidly secured by means of a rivet to the cursor iN.

Over the cursor iN and over its scale a is rotatable another lozenge shaped transparent cursor iU, carrying two lines, namely:—

a longer line U destined to indicate the values of the underlying scale As, and a shorter line, $\frac{M}{P.L}$, which is destined to indicate these values on the scale a.

The line $\frac{M}{P.L}$ indicates the value 12, when U and iN are coincident. Therefore, if the binding condition is different from 12, it is sufficient to rotate the line $\frac{M}{P.L}$ from the value 12 to the desired value, for instance 10, 8, and so on, and then to read the number of stirrups under the line U instead of iN.

Moreover, the cursor iU enables one, by moving the line $\frac{M}{P.L}$ to 4, to read the stirrups of a single triangular diagram for the case of dissymmetrical binding conditions on the rest, by carrying subsequently on M the sum of the positive moment and of the negative moment, to either one or to the other end. Although the algebraic explanation is deemed unnecessary, it is sufficient to indicate that when the line $\frac{M}{P.L}$ is placed on the number 4, the hypothesis of the most frequent binding case 12 as well as the hypothesis of symmetrical binding are removed, thereby obtaining the possibility of readily reading on the device the number of stirrups in all possible cases.

6. Particular Composition of the Scales r, t and k.

The values r, t and k, are dependent upon n, fc and fs according to the formulæ (1b), (2b) and (3).

For sake of clarity, heretofore it has been assumed that the rings r, p and s, are provided with common logarithmic scales whereon it is possible to read the resulting values r, t and k, (without the necessity of calculating the formulæ (1b), (2b), (3) from suitable tables, as commonly used) when said values are read in register with the desired values of n, fc and fs.

It is known in the common slide-rules for reinforced concrete, to avoid the operator having to search the tables for the values of r, t and k by arranging on the logarithmic scale of the values r (related to a predetermined n and to a predetermined fs) an indication of the values of fc corresponding to the value of r concerned with the given position. Thus there are scales with a whole succession of fc values (for instance from 10 to 100 Kg/cm^2, or from 250 to 2500 psi) instead of the relative factors r (and analogously for t and k).

The aforesaid feature is a very useful one in the common slide rules for reinforced concrete but is negatived since at the most it is possible on the slide rule to have one scale for r, one for t and one for k, i.e. it is possible to read all the values of fc for one value of fs and for one value (or at the utmost two values, by the two surfaces of the slider) for n. Even if it is possible to arrange two or three scales instead of one scale, the result is always far removed from the 20 to 25 scales which would be necessary. This fact compels use of the tables for the frequent practical cases varying between $fs=2400$ Kg/cm^2 and $fs=800$ Kg/cm^2. On the contrary, the present device allows for the easy and clear search not only in the indicated field, but also under 800 Kg/cm^2 and down to 100 Kg/cm^2, with a result which is particularly suitable for solving, according to a very easy system, the problems concerning the combined compressive and bending stresses.

Thus, in the annular space for the scales *r* or *t*, nine or only seven scales are arranged; each of these nine or seven scales does not differ from what has been previously described, except for an exaggerated extension of the values of *fc*. Each of said scales is therefore traced for a single value of *fs*, but it serves, at it will be hereinafter described, for three values, whereby nine scales give twenty seven values for *fs* (only twenty four of which are utilized), or seven scales give twenty one values of *fs* (only twenty of which are utilized).

This possibility is explained by a perusal of the formula giving *k*, *r* and *t*.

The formula (3) $k=\dfrac{n}{n+\dfrac{fs}{fc}}$, shows that the value *k* does not change if both *fs* and *fc* are multiplied by the same number. This property may be easily exploited if, as common factor, either two or one-half is chosen. In fact the scale of $fs=1000$ Kg/cm^2 will be read as scale of $fs=2000Kg/cm^2$, if the numbers of *fc*, thereon arranged, are mentally doubled, by an easy operation, and likewise said scale will be read as scale of the values *k* for $fs=500$ if the values read thereon will be mentally halved.

A table *k* in Figure 6 (discounting two small numbered scales F^1 and *Cr* arranged at the upper and lower edges respectively, to be described hereinafter) has nine scales of the values *k* for the following nine suitably chosen values of *fs*: 2500, 2400, 1400, 1100, 1000, 900, 800, 300, 200. On each of said scales, the value *fc*, arranged at the place of the concerned value *k* are not limited to the range 100—20 Kg/cm^2, which can be widely considered the used values, but are extended to double the maximum on one side and half the minimum of the other side, i.e. from 200 to 10 Kg/cm^2.

According to the above, by doubling or halving the values of *fc* read as for a halved value of *fs*, i.e. on the nine indicated scales, it is possible to mark immediately (by either of the two lines of a transparent cursor *ik*) the values *k* of all of the following scales:—

2800 (1400×2), 2500, 2400, 2200 (1100×2), 2000 (1000×2), 1800 (900×2), 1600 (800×2), 1400, 1250 (2500×½), 1200 (2400×½), 1100, 1000, 900, 800, 700 (1400×½), 600 (300×2), 550 (1100×½), 500 (1000×½), 450 (900×½), 400 (800×½), 300, 200, 150 (300×½), 100 (200×½).

On the transparent cursor *ik* these values of *fs* are marked in register with the line to which they are related and they are divided in three columns headed ½, 1, 2, in order to advise the operator that he has, according to the particular case, to halve, to leave unchanged or to double the values of *fc* read on the concerned line for a determined value of *fs*. The process is quick, easy and reliable.

Therefore, the table *k* with its cursor *ik* can easily find the positions for all the pairs of values of *fc*/*fs* in the range *fc*: 100—20 and for the aforesaid twenty four values of *fs*. For any of these positions the cursor, on a common scale arranged at the bottom, would show the relative values *k*, but this is not necessary in that in the central zone in the ring *s* there is no common scale for the values of *k* but there are two scales of values of *k*, shown by the values of *fc*, concerning $fs=1400$. There are two scales, the first being for $n=10$ and the second for $n=8$. Thus, when the value $fs=1400$ is used, as frequently occurs, no need arises for the table *k*; on the contrary, when different values are used, the cursor *ik* of the table *k* enables the value of the scale 1400 to be found corresponding to the desired value *fc*/*fs*. This value is marked by the line of the cursor *is* on the scale *k* of the circle *s*, for $n=10$ or for $n=8$ according to the operator's need.

The system used for *r* and for *t* is similar except for some slight difference.

By examining the algebraic expressions of the two aforesaid values:—

(1b) $$r=\sqrt{\frac{2}{fc.j.k}};$$

(2b) $$t=\frac{fc}{2.fs}\sqrt{\frac{6n}{3\ fs+2\ n.fc}}$$

it can be seen that, for a given value of *n*, if the variables *fc* and *fs* are multiplied by the same factor (2, ½), (bearing in mind that in this case *k* is constant), both *r* and *t* will be divided by $\sqrt{2}$ or $\sqrt{\tfrac{1}{2}}$, respectively.

The arrangement of the said transparent cursors *ir* and *it* is based on this remark. Said cursors are provided with a simple central line whereby the *fc* values are read, as they are marked, which concern the values *fs* which are engraved on the plate laterally to this central line and in correspondence with the several circles 2400, 1400, 1000, 900, 800, 300, 200.

On the same transparent cursors *ir* and *it*, there is also a second double radial line, i.e. formed by two lines close to one another, displaced with respect to the first line through a logarithmic angle equal to the square root of two, in the same direction as the scales *r* and *t*, i.e. to the right for *r*, and to the left for *t*. This line is a double line in order to suggest to the operator that the values of *fc* marked on the circles fs shown by the numbers marked laterally to said line must be doubled, and that, corresponding to the same circles and the other line, said figures have the values 2800, 2000, 1800, 1600, 600, 400 (disregarding the value 4800 which is not utilized).

Also a third radial line is present on said transparent cursors, and said line is a dotted line displaced with respect to the central line by a logarithmic angular amount equal to $\sqrt{2}$, but in the opposite direction to the other line, in order to divide the values r and t by $\sqrt{\frac{1}{2}}$. The dotted line serves to remind the operator that he must mentally halve the values of fc marked by said line, which relate to the values of fs shown by the near engraving corresponding to the several circles and which are in this case: 1200, 700, 500, 450, 400, 150, 100.

Thus, with only seven circular scales, it is possible to have the readings of the scales fc from 100 to 20 for all the following values of fs: 2800, 2400, 2000, 1800, 1600, 1400, 1200, 1000, 900, 800, 700, 600, 500, 450, 400, 300, 200, 150, 100. In this case it has been deemed convenient to omit the values 2500, 2200 and 1100 in order to obtain seven broader scales instead of nine. By nine scales it would have been possible to have all of the values shown for k.

For values of n different from $n=10$ on the scales r and t, other rings with the same number of scales (either seven or nine) prepared for the different values of n used, which are two or at the maximum four, are applied on said scales by slightly lifting the cursors ir and it.

7) Reductions in the Width b and in the Taut Reinforcement As Due to Insertion of an Amount of Compressed Reinforcement As^1—Rapid Scanning Obtained by the Particular Use of Two Scales Termed: "by F^1" and "by Cr".

When the dimensions d, b, As and so on, have been found for a cross-section of an inflected beam or of a beam compressed and deflected, for any value of n, fc and fs, it is possible by keeping these values dependent upon the resistance of the materials, to reduce the width b by a certain amount b^0 for the the same height d, if in the compressed portion, a certain section of iron As^1 is inserted spaced apart by a certain distance d^1 from the cross-section of the taut iron As should be extreme compressed limb. Therefore, the cross-section of the taut iron As should be slightly reduced.

It is a common practice, in this case, to assume that the previously found cross-section of the beam, having the height d, a width b, and the taut iron As, is divided by a cut at right angles to the width into two sections which, for sake of clarity, are assumed to be unequal. It is as if two narrow beams were substituted for the single beam, said two beams being submitted to equal unitary stresses on the materials, the height of the two beams being equal to d, whilst the greater beam has the width b^1 (less than b by an amount b^0) and the other and smaller beam has the residual width b^0. Also the area of the taut iron As can be assumed to be divided into parts proportional to the two portions having the widths b^1 and b^0, respectively. The portion corresponding to the minor section having the width b^0 will be termed A^0.

This imaginary division has been performed in order to remove the concrete from the smaller section, substituting for it a resistant torque formed only by areas of taut iron A^0 and compressed As^1, spaced apart by the distance $d-d^1$, which, in fact, are incorporated within the greater concrete beam having the width b^1 smaller than b.

From the common knowledge of the equilibrium of the inner stresses of a section of a reinforced concrete, it can easily be obtained that, if the distance d^1 of the compressed outer limb where the compressed iron As^1 is to be placed is fixed with respect of d by a certain ratio which will be termed g ($g=\frac{d^1}{d}$), then between the portion of taut iron A^0, pertinent to the section which has been termed "the smaller section" and the area of compressed iron As^1, the following relation exists:—

$$As^1=A^0.\frac{1-k}{k-g}.\frac{j}{1-g} \qquad (7)$$

Thus it is possible to obtain As^1 from A^0 by multiplying this latter term by a certain coefficient depending upon k and upon the ratio g. If, for instance, the value of this ratio is fixed at 0.07, it is easy to obtain for any value of k a coefficient which when multiplied by A^0 will give As^1; this coefficient will be termed $\frac{1}{F^1}$, and according to equation (7) we will obtain:—

$$\frac{1}{F^1}=\frac{1-k}{k-0.07}.\frac{j}{1-0.07} \text{ and } As^1=\frac{A^0}{F^1} \qquad (7^1)$$

Furthermore, it is easily found that, when the area of compressed iron As^1 has been substituted for the concrete of the smaller section, due to the raising of the centre of gravity of the compressed zone (which has been substituted by a smaller area of iron) the taut iron A^0 which was formerly necessary for this portion will be reduced by a certain amount which will be termed Cr, according to the equation:—

$$Cr=A^0(\frac{1-j}{1-g}) \qquad (8)$$

Thus, if for g a fixed value equal to 0.07 has been chosen, for any value of j i.e. for any value of k (as $j=\frac{1-k}{3}$) a coefficient may easily be calculated, which will be termed q, and according to equation (8):—

$$q=\frac{1-j}{1-0.07}, \text{ and } Cr=A^0\ q \quad (8^1)$$

These formulæ have been studied for obtaining the two original logarithmic scales terms "for F^1" and "for Cr" which, when inserted in this device give this latter the original feature of instantaneously passing from any section without compressed iron to any other section having a lesser width, indicating at the same time the relative amount of iron to be inserted in the compressed zone, and simultaneously the correction for the taut iron. This occurs as follows: the logarithmic scale a has been described, said scale being rigidly secured by a rivet 76 to the transparent cursor iN, which is rotatable with respect to the cursor ip. Also, the use has been described both of the cursor iN and the logarithmic scale for reading, on the scale As, the number of stirrups Av. Examining Figure 6, it will be seen that the scale a is not rigid with the cursor iU, which, due to its lozenge shape, is clearly shown in Fig. 6. The scale a is rigid with the cursor iN which, as shown in Fig. 1, lies beneath the cursor iU, the line of which is clearly shown at Fig. 6 together with the close engravings of the symbols N, U. The cursor iN with its line NU may be rotated, with respect to the cursor ip and to its line, through a certain logarithmic angle which is shown by the inner end of the line ip on the small logarithmic scale a, which revolves together with iN. On the circular scale s and in the table k is arranged a scale of the values of F^1 corresponding to the near values of the scales fc/fs of the ring s or of the table k.

Now, if the inner end of the line ip is caused to mark the aforesaid value on the small scale a, the line NU on the scale As will mark a corresponding angle in diminution, so that for each value As marked by the line ip, Nu will mark the value $As\times 1/F^1$. The equations (7) and (7^1) show that this latter is the value As^1 when on As is read the value which has been termed A^0.

When the line iN is arranged as aforesaid with respect to ip, it is assumed the device is set with all its scales at the places requested in order to have the first basic section of the beam. In other words if the scales r, t and s are on the desired values fc/fs, if iM is on the value M requested by the problem, if the line ip shows, besides the values kd, the values d, b and As, as aforesaid, if b is to be diminished by a certain amount b^0, and simultaneously the area As^1 to be inserted in the compressed zone, at a distance $d^1=0.07\ d$ from the compressed fibre, is to be ascertained, the knob 1 is rotated, without pressure until on b is read the width b^0 by which the original width is to be diminished. In this condition, without need to read them, the two scales have been arranged with the values pertinent to the smaller of the two portions into which the original section has been assumed to be divided. N will mark the portion of bending moment supported by this portion of the beam, k remains on the same indication (such as r and t) the value b^0 appears for the value b, and on As there will be shown the concerned portion A^0. Only b^0 will be read among these factors These latter have been cited only for demonstrating that at the same time, the value $A^0\frac{1}{F^1}$, will be read under the line NU, this latter value being that of the value As^1 which was sought.

Therefore, by slowly revolving the knob 1, while the line ip shows on b all of the increasing values of b^0, the line NU will simultaneously show on the scale As all the related values of As^1, allowing a scanning operation which would be quite impossible by other means.

As aforesaid, in this operation it is unnecessary to read A^0, as this value which has been initially deducted from the original value of As, is then inserted in the novel section which has reduced width, but the original iron strength is due to the insertion of both the taut iron A^0 and compressed iron As^1. Therefore the original area As is formed again for the taut iron. As aforesaid, both the amount A^0 and consequently As can be diminished by a certain amount Cs depending upon the fact that compressed iron has been substituted for the concrete of the smaller portion. This amount is given by equation (8^1) as a function of A^0. On the other hand, equation (7^1) shows that $A^0=As^1.F^1$, and therefore equation (8^1) can be written:—

$$Cr=As^1.F^1.q \qquad (8^{11}).$$

On a scale of Cr are collected the values $F^1.q$ which, according to the above, result from the expression:—

$$F^1.q=\frac{k-0.07}{1-k}.\frac{1-0.07}{j}.\left(\frac{1-j}{1-0.07}\right) \quad (9)$$

These values depend exclusively upon the value of k and upon the value j obtained therefrom, so that this scale Cr is placed adjacent to the lower part of the table k and on the circle s so as to serve (as does the scale F^1) for all the pairs of values fc/fs which have the value k in common.

Then, after having turned the cursor iN so that the inner end of the hair line on cursor ip shows on the scale a the number marked on the scale F^1 either by the cursor is or the cursor ik, the operator turns the cursor iU so that the line U marks, on the small scale a, the value $F^1.q$ which is easily read by the operator on the scale Cr and is marked by the same line of either cursor *is* or *ik*, which the operator used for reading F^1. In this position, the end of the same line U shows on the same scale A*s* an amount corresponding to $F^1.q.As^1$, which, according to equation (8^{11}) is exactly the diminution C*r* to be applied to the area of taut iron A*s*.

Thus, by the aforesaid scanning the operator, having suitably arranged the two cursors *i*N and *i*U according to the figures easily read on the scales F^1 and C*r*, can instantaneously see, for all of the values b^0 of the reduction of the width, all the values As^1 of the area of compressed iron and all the values C*r* of the reduction of area of the taut iron As^1.

8) Scanning of the Height Reductions Due to the Presence of a Certain Amount of Compressed Iron, Expressed as a Percent of the Taut Iron A*s*.

From the known formulæ giving the height *d* and the taut reinforcement A*s* of a beam, when a certain amount of compressed iron K.A*s* is placed in the centre of gravity of the compressed zone, it is easy to obtain an expression which will be termed:—

$$u = \frac{fs}{fs - 2/3\ n.K.fc} \qquad (10)$$

such that:—

$$d = r.\frac{M}{u.b} \qquad (11)$$

$$As = t.\ \sqrt{M}\,.\,\sqrt{u.b}. \qquad (12)$$

The formulæ (11) and (12) are identical, even if written in a different form, to the formulæ (1^1) and (2^1), however the product of the width *b* with the coefficient *u* has been substituted for the width *b*. Then, in the ring *u* (Figs. 1 and 6) are marked five scales for five values of *n* and each scale carries, for four values of *fc*/*fs* the values of *u* calculated according to the expression (10), as shown by the figures 0, 2, 4, 6, 8, 1/1 representing 0%, 20%, 40%, 60%, 80%, 100%, i.e. by the value K, equal to $\frac{As^1}{As}$ on which they depend.

A transparent ring *i*K is rigidly secured by rivets 76 to the scale *b*, said ring being provided with five lines which are normally over the five zero values of the five scales *u*.

It will now be assumed that the scale *b* and the elastic ring 28 (Fig. 1) are released from frictionally holding said elements stationary in the box 27, and the elastic ring 28 is tightened on the ring 29 secured to the cursor *ip*.

As aforesaid, when the knob 47 is acted upon when in its pressed condition, all the scales remain stationary while the cursor *ip* revolves. Thus scanning is obtained of all the possible solutions *d*, *b* and A*s* corresponding to a predetermined desired value of M and to the conditions *fc*/*fs* marked on scales *r* and *t*.

If the same actuation is carried out while the elastic ring 28 is tightened on the ring 29, the scale *b* also revolves, with the cursor *ip*. If both cursor *ip* and scale *b* are revolved together through a certain amount *u*, concerned with the value K shown by the transparent ring *i*K on the scale *u*, cursor *ip* will mark on scale *b* the same value as before, but on scales *d* and A*s* will be marked the values related to a width *u*, *b* which is also, according to equations (11) and (12) the value of the width *b* if a certain amount of compressed iron As^1= K.A*s* is placed in the centre of gravity of the compressed zone, instead of marking the values related to the width *b*.

Therefore it is possible, in this position of the elastic ring 28, by the slow movement of cursor *ip* and the simultaneous movement of plate *i*K, while the value *b* of the width remains stationary, to scan all the variations in height of the area A*s* due to the variations of the percentage K of the compressed iron relatively to the taut iron A*s*.

The mechanism of the central pivot is contained in the box 27, and it is used for allowing scale *b* to remain stationary in the general base, or to displace it with the index *ip* over by scale *u*. In Figure 7 the mechanism is shown in exploded fashion. Figure 7 shows the box 27 in section and within which is mounted the elastic ring 28. The height of the ring 28 is slightly less than that of the vertical wall of the box 27 and a sector A—B has a much reduced height, so as to leave the place for a lever 53 and a wheel 54. In the sector A—B the ring 28 is cut by a Z-shaped cut located under the lever 53. Two small pins 55 enter into holes in the lever 53, said lever having at its end a plate spring supporting the wheel 54. In its remaining portion, the ring 28 has a set of vertical cuts, which reach only part way therethrough, in order to increase its elasticity, said elasticity keeping it adherent to the wall of the box 27. The wall of the ring 28 also has a horizontal rectangular slot 83 passing therethrough and in the middle of the slot 83 there is a vertical hole 84 passing through the portion below the slot, a ball 61 of the same diameter being located in the hole 84. The ring 28 has a vertical hole in its upper horizontal surface to receive a pin 79 projecting from a ring 30 secured to the scale *b* and the transparent plate *i*K.

A ring 57 has its internal diameter corresponding to the external diameter of the box 27 around which it is disposed and relatively to which it can rotate. The ring 57 carries an internal projection or slider 59 which penetrates into a suitable cut or slot 81 in the vertical wall of the box 27, such slot 81 having a greater length than the slider 59. Through said slot, the projection or slider 59 of the ring 57 reaches the internal surface of the box 27.

In the base of the box 27, below the hole

84 of the ring 28 when same is in a pre-determined position, which can be called the "zero" position, there is a depression 82 having a diameter corresponding to the hole 84. The ring 57 also has a horizontal pin 60, which passes through the slot 81 of the box 27 and the slot 83 of the ring 28.

A pawl 58 mounted on the ring 57 reaches the level of the graduated scale *u*. The displacement of the pawl 58 imparts rotation to the ring 57 to cause movement of its slider 59 against the wheel 54, which is thus urged inwardly. Owing to this movement of the wheel 54, the elastic ring 28 shrinks, releasing itself from the box 27, and pressing upon a ring 29 disposed within the ring 28. The ring 29 carries on its upper edge four teeth engaging recesses in the edge of the transparent plate *ip*, whereby the ring 29 and plate *ip* move together.

Movement of the plate *ip* with the scale *b* (connected to the ring 28 by means of the pin 79 of the ring 30) can thus be obtained by elements which do not prevent the whole rotation of the plate *ip*, and such interlocking will cease automatically when the scale *b* goes back to its basic position, i.e. when the index lines *i*K mark zero in the scales *u*, locking in this position to the steady base.

When the operator displaces the pawl 58 to the right, e.g. with a pencil point, the slider 59 of the ring 57 pushes the wheel 54 inwardly and, in consequence, the elastic ring 28 and the scale *b* rigid therewith are disengaged from the stationary box 27 and remain connected to the plate *ip* through the ring 29, following said plate in its rotational movement. When the operator displaces the pawl 58, to push the guide block of the ring 57 against the wheel 54, the pin 60 of the ring 57 moves from the lefthand wall of the slot 83 to the right-hand wall, passing over the hole 84. The small ball 61 which is disposed within the hole 84 does not prevent the passage of the pin 60 because in that position the ball is below the upper surface of the slot, projecting into the depression 82.

When the operator, with the central button 47 of the machine pressed, rotates the wheel 22 (Figure 4) the plate *ip* displaces with it, towards the left (because this is the direction of the scales *u*), the scale *b* with the plate *i*K, and with the ring 28. The right-hand wall of the slot 83 of the ring 28 pushes the pin 60, forcing the ring 57 to rotate together with the ring 28 and thus keeping the slider 59 against the wheel 54, with the ring 28 therefore always locked to the ring 29. Said rotational movement of all the portions takes place without the slider 59 engaging the end portion of the slot 81 for this has a sufficient width to allow for displacement corresponding to the greatest width of the scales *u*.

As soon as said movement has begun, the ball which was within the hole 84 of the ring 28 leaves the depression 82 and will project a little into the slot 83. In this manner, when the operator begins the return movement towards the right, the projection of the ball will cause the pin 60 and the ring 57 to follow such return movement, preventing any relative movement of the pin 60 and the ring 57 which might free the pressure of the slider 59 against the wheel 54 before the due time. However when the basic zero position of the scales *u* is reached, the hole 84 will be in alignment with the depression 82, the ball 61 which is inside the hole 84 will descend a little, and the pin 60 will go back, freeing the pressure of the slider 59 on the wheel 54. In consequence the wheel 54 will go forward, within the slot 81, allowing the expansion of the elastic ring 28, which will free the ring 29 (and the index *ip*) and will engage by its outer periphery with the box 27, thus fixing the scale *b* to the same in the basic required position.

9) Brake

An important mechanism, even if an accessory, for the operation of the device hereinbefore disclosed is formed by a braking means acting when the knobs 1 or 46 are rotated under pressure.

Under these conditions, the parts released by the action of the springs 4 or 35 must be stationary together with the female cones 6 or 34.

When the knob 1 is pressed, only the wheel 3, the bearing 8, the cursor *ir*, the wheel 14 and the part 17 together with the scales carried thereby have to move. Moreover, movement of the knob 46 must move only the wheel 37, the bearing 38, the cursor *it*, the wheel 42, the bearing 43 and the scale A*s* carried thereby. Now, it is possible that the parts which should be stationary might move slightly due to the small friction between the shafts fitting in one another.

In order to avoid this drawback, a metallic collar 62 is arranged under the knob 1 and a similar collar 63 is arranged under the knob 46. During the two operative steps as aforesaid, the knob 1 or the knob 46 must be pressed to release the cone 5 or the cone 78, respectively. In these steps, the collar 62 presses a small spring 64 against a rubber packing 66 which is contained between said spring 64 fixed to the box and nuts 68 and prevents these latter from moving and therefore also prevents movement of those parts connected to the bushing whereon said nuts are screwed.

Analogously the pressure of the knob 46 urges the collar 63 against a spring 65 pressing a rubber packing 67 against nuts 69, obtaining the same braking effect during the revolution of the knob 46 when pressed.

10) Calibration Means

As aforesaid at the beginning of this specification, the unitary mounting of accurate logarithmic scale on toothed wheels which are parts of gears, the relative positions of which are established by the meshing needs of the teeth, has been rendered possible by means of calibration means.

These devices are formed by the friction cones 10 and 40 and by the respective female cones carried by the pieces 11 and 41. These latter are pressed against the cones 10 and 40 by nuts 70 and 71.

The nuts 70 and 71 can be loosened by means of special keys which are owned only by the manufacture of the device. When the device has been assembled, the two nuts 70 and 71 are loosened and the knob 1 is rotated without pressing so that the line *iM* shows a pre-determined precise value on the scale M. Then, the same knob 1 is pressed to rotate the scales *d*, *kd* and *s*; the scales *kd* and *s* having already been arranged in their exact positions with respect to scale *d*. The scale *d* is then rotated so as to align a pre-determined position of this scale with a pre-determined position of scale *b* so that the numbers according to a precise calculation according to the aforementioned formulæ are set up.

Analogously, the knob 46 is pressed and rotated and the same A*s* is arranged so that the value according to the aforesaid basic calculation is aligned with the prescribed values of the scales *b* and *d*. Whilst holding this position of all of the scales stationary, the released female cones of the pieces 11 and 41 are freely turned so that cursors *ir* and *it* exactly mark on the scales *r* and *t* the value of *fc*/*fs* which has been selected for the basic calculation.

When the nuts 70 and 71 have been tightened in this position, the scales will remain in the precise corresponding positions which, depending upon the said equations, will be repeated for any other combination of values with the same accuracy as the first position and with the accuracy afforded by the angular displacements of the pieces and by the manufacture of the logarithmic scales.

11) THE PARTICULAR ARRANGEMENT ALLOWING THE BENDING MOMENTS *M* TO BE READ EITHER IN " LBS. INCHES " OR IN " FT. KIPS " FOR THE COUNTRIES USING THE BRITISH METRIC SYSTEM

The cursor *iM*, shown in Fig. 4, is provided, for the countries using the MKS system, with a single central hair line, shown in Fig. 6. This line is sufficient for reading on the underlying scale the bending moments, either expressed in Kg.—cm. or in Kg.—m.

A simple constructional contrivance completes the embodiment of the device hereinbefore described for countries using the British metric system. For these countries, the single line shown in Fig. 6, is replaced by two radial lines, spaced apart by a logarithmic circular sector 12 taken on a double base (base 180°).

Although this arrangement is not shown in the figures, as far as the numbers of the scale *M* shown at Fig. 6 are concerned, it is clear that the spacing of said two lines will be equal to the spacing between the two numbers 200,000 and 240,000. The cursor *iM* is engraved adjacent the left-hand line with the indication " lbs. inches " and adjacent the right-hand line with " ft. kips ".

Thus the operator can, without taking into account the factor of 12 which, besides the zero, must be considered in the concerned formulæ, easily set the moment in " ft. kips " in register with the suitable line, and read the values of *kd*, *d*, *b*, and so on in inches, as required.

On the contrary, if the moment is expressed in " lbs. inches " the operator uses the other line, i.e. the left-hand one.

WHAT I CLAIM IS:—

1. Apparatus for the solution of a plurality of equations such as those used in calculations relating to reinforced concrete structures, comprising a circular logarithmic slide rule, with additional logarithmic scales and respective co-operating indices rotatable about axes separate from the axis of the main slide rule, both the additional scales and their indices being coupled with certain respective scales of the main slide rule so as to rotate with them, and including selectively operable coupling devices for coupling each additional scale to its respective index member for rotation together.

2. Apparatus for the solution of a plurality of equations in calculations relating to reinforced concrete and of the type in which there are more factors appearing in the equations than equations connecting such factors, the solution being based upon selecting in any way a number of factors equal to the number of equations as factors for which values are required, i.e. unknown factors; selecting a further factor as a variable factor to which a range of values is to be given and selecting for all the other factors fixed values, such apparatus comprising a plurality of circular logarithmic scales mounted on at least two axes, one scale for each factor; revolving scale supporting means for certain scales; revolving or fixed index supporting means for each scale for reading on the scale relative displacement of the scale supporting means and the index supporting means; means to set up upon the relevant scales the values assigned to each constant factor, the scales and the indices being interconnected by mechanical connecting means so that those scales which represent factors which are functions of any factor to which a value has been assigned are displaced (when the scale representing such assigned value factor is displaced) relatively to their indices and by an amount in accordance with the equations whilst those scales which are not functions of such factors are caused to

remain constant with respect to their indices; and calibration means for setting up the original relative positions of certain of the scales and their scale supporting means independently of the said mechanical connections.

3. Apparatus according to Claim 2, characterised in that the mechanical connections between the scale supporting means and the index supporting means comprise toothed wheels, the gear ratios of which automatically take into account the necessary variations in any factor which is a function of any other factor.

4. Apparatus according to Claim 3, characterised in that the scales are supported by circular mechanical supports formed by concentric hollow shafts and co-axial toothed wheels distributed on a convenient number of axes, each scale being concentrically fixed to its own mechanical support and the associated toothed wheel, the relative position of the scale and the wheel-teeth being variable and the calibration means being provided for aligning the scales on their required relative position independently from the relative displacements of the teeth of the wheel to which said scales are fixed.

5. Apparatus according to Claims 2, 3 or 4, characterised in that said calibration means comprise friction cones, said cones when disengaged allowing for the arrangement of all the logarithmic scales in an accurate original position according to a pre-determined computation, without movement of the mechanical connections.

6. Apparatus according to any of Claims 2 to 5, characterised in that the index supporting means are normally connected to their associated scales by means of friction devices, independently of the relative position in which they have been placed by the operator, said connections being broken by the operator for those scales on which he intends to operate and being automatically re-established as soon as operation on said scales is ceased.

7. Apparatus according to Claim 6, characterised in that the connection between the scales and the indicating means is obtained by an appropriate degree of pressure between co-axial scale supporting plates and index supporting plates, the relative area of the contact surfaces and their co-efficient of friction determining the required friction effect.

8. Apparatus according to Claim 7, characterised in that the connection between scales and indicating means is obtained by the action of a spring on interfitting cones, one of which is fixed to the scale, while the other is fixed to the index, so that by passing on a driving knob the separation of the cones is effected and in this last position it is possible to rotate the group of scales fixed to one of the cones with respect to the group of scales fixed to the other cone.

9. Apparatus according to any of Claims 2 to 8, characterised in that it comprises braking means for increasing the friction between the stationary parts of the machine and the scales which are to remain steady when others are rotated, such means being brought into action by the axial displacement against a spring of a knob for causing such rotation, the action of the braking means ceasing therefore as soon as the knob is released.

10. Apparatus according to Claim 9, characterised in that said braking means are formed by collars arranged under the rotatable knobs, said collars (when the associated knob is lowered) engaging elastic elements fixed to the frame, which then increases their friction with the parts connected to those wheels which must remain stationary, locking such parts in the position occupied previous to the operation of the knob.

11. Apparatus according to any of Claims 1 to 10, characterised in that a scale indicating bending moments is provided with two spaced reference lines adapted to allow a problem to be set in units differing from those of the solution without the need of introducing a conversion factor different from 10^n into the calculation.

12. Apparatus according to any of Claims 1 to 11, characterised in that a first main plate carries numbers corresponding to the diameter of the iron which is intended to be used for the vertical reinforcements of the concrete structure, said numbers being arranged at such angular logarithmic distances from a base line that a second transparent plate, rotatable over the first plate, shows by its reference line the number of the necessary reinforcements, having the desired diameter, when the base line shows other dimensions of the section.

13. Apparatus according to Claim 12, characterised in that a third transparent plate, over the second plate, can turn on said second plate with which a scale is rigid, said third plate being provided with two divergent reference lines whereby it is possible to read the logarithmic scale carried by the second plate, said two reference lines being divergent through an angle corresponding to the same value assumed in the base conditions, one or other of said two reference lines measuring the angle subtended by it and the main reference line of the first plate.

14. Apparatus according to Claim 13, characterised in that it comprises scales adapted to provide the numerical data which, shown on the scale of the second rotatable plate, allow the second plate and the third plate to be arranged at such an angular divergence that while a main pointer reads the amount of width which is to be subtracted from the section, the index lines of the second and the third plates show the corresponding amount of iron to be added in the compressed zone and

the amount of iron to be subtracted from the taut iron.

15. Apparatus according to Claim 14, characterised in that transparent cursors are provided for setting in the computing device amounts depending upon the stresses in the iron and the concrete cursors being provided with three different index lines (for simple, double or halved indications) which are mutually divergent through a logarithmic angle equal to the square root of two, so that each scale can be used also for a doubled value or for a halves value, thereby to provide a triple range of the values shown on the scales adjacent the lines.

16. A logarithmic computing device according to Claim 15, characterised in that it comprises means adapted to connect at will the width scale to the index line in order to move this latter through amounts shown on an adjacent scale and concerning the insertion of predetermined amounts of compressed iron, expressed as functions of the taut iron, said means being controllable without preventing the overlying index from complete revolution and being automatically released as soon as it is returned to zero.

17. A logarithmic computing device substantially as hereinbefore described with reference to and as shown in the accompanying drawings.

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Leamington Spa: Printed for Her Majesty's Stationery Office, by the Courier Press.—1960.
Published at The Patent Office, 25, Southampton Buildings, London, W.C.2, from which copies may be obtained

831,066 COMPLETE SPECIFICATION
5 SHEETS
This drawing is a reproduction of the Original on a reduced scale.
SHEET 1

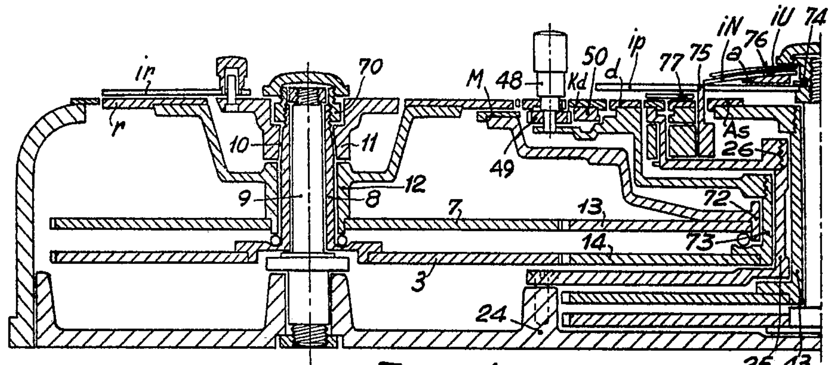
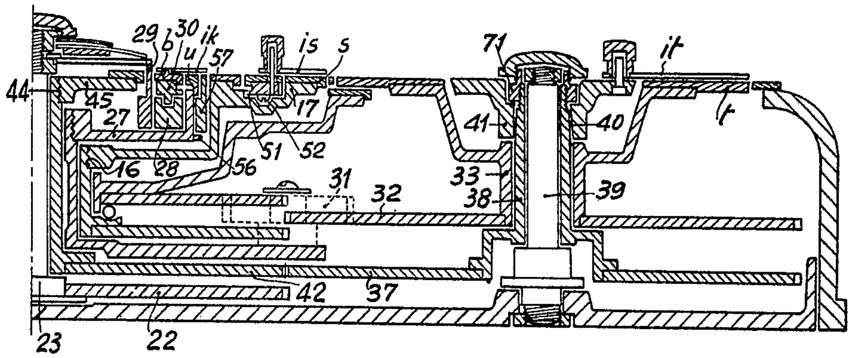


FIG. 1



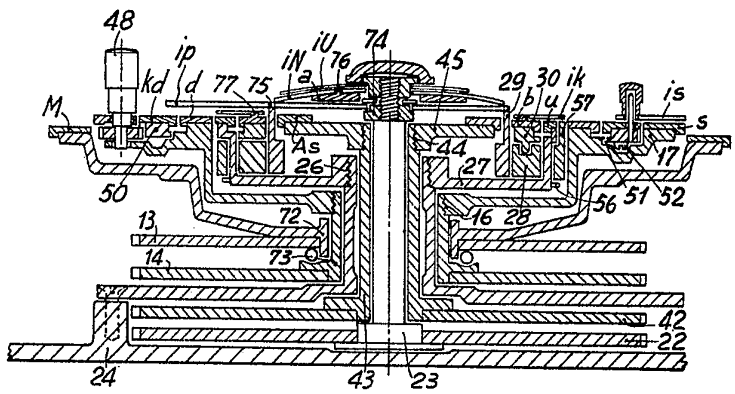


FIG. 1b

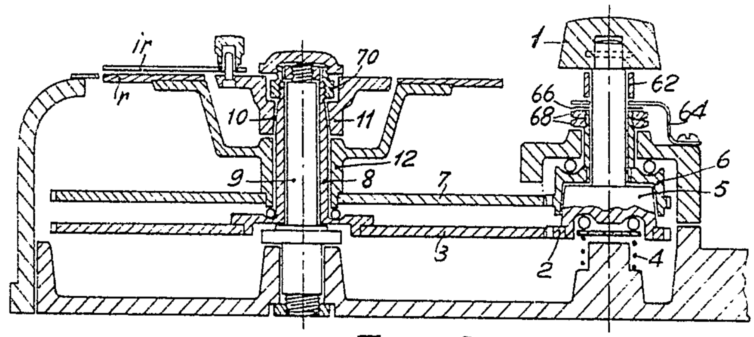


FIG. 2

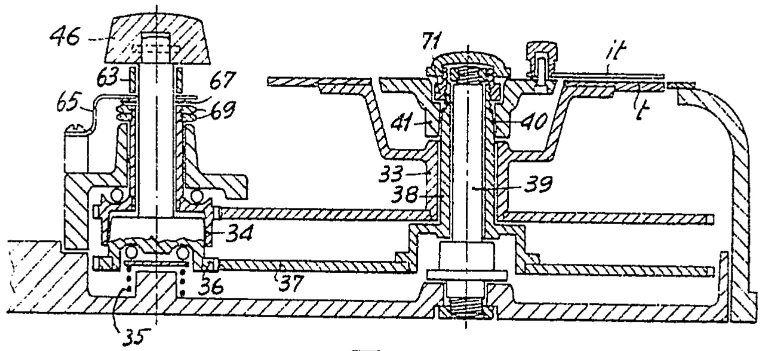


FIG. 3

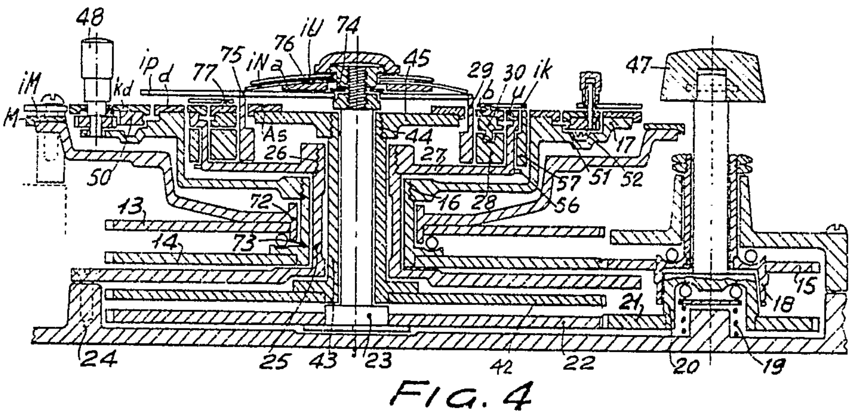


FIG. 4

831,066 COMPLETE SPECIFICATION
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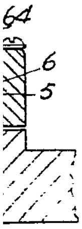
SHEETS 2 & 3



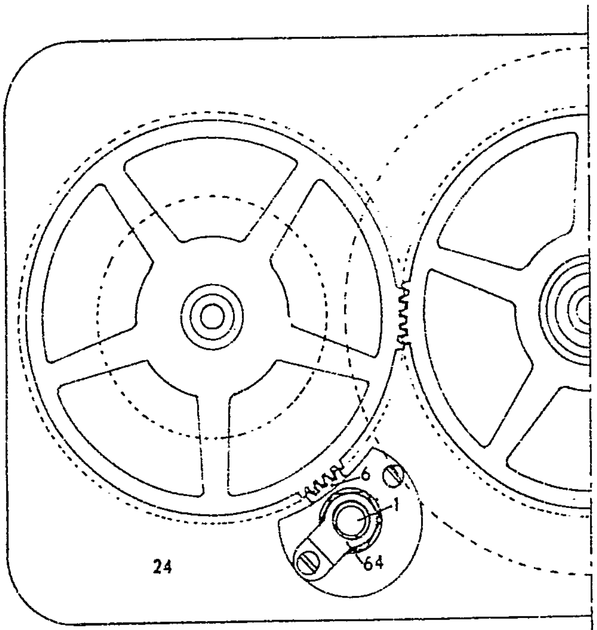
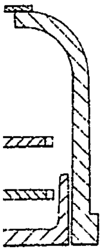
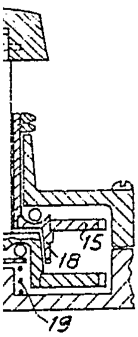
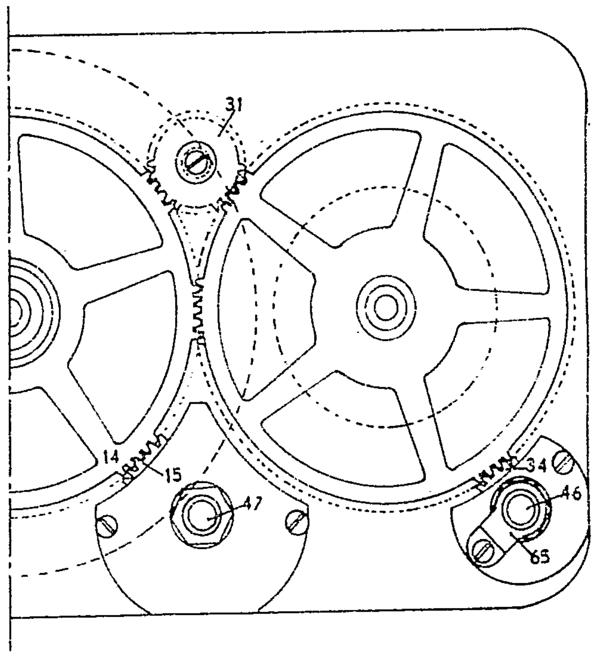


FIG. 5







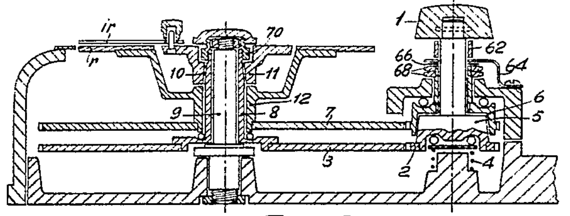


FIG. 2

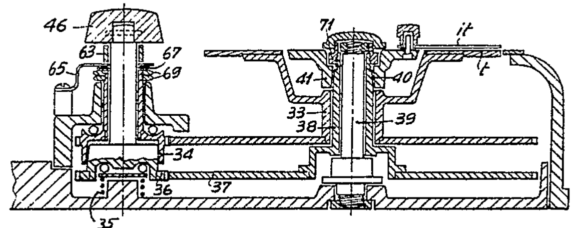


FIG. 3

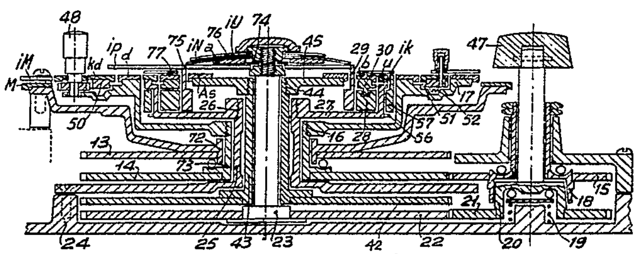


FIG. 4

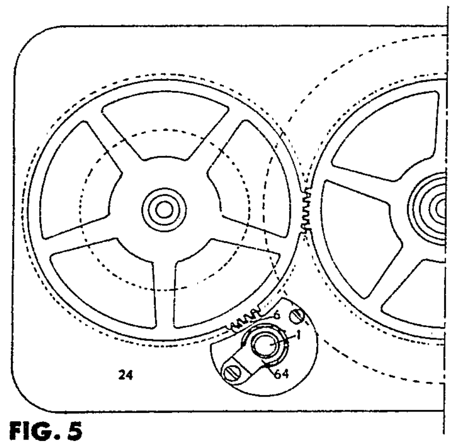
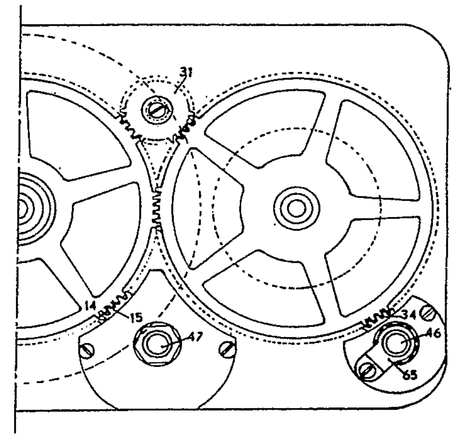


FIG. 5



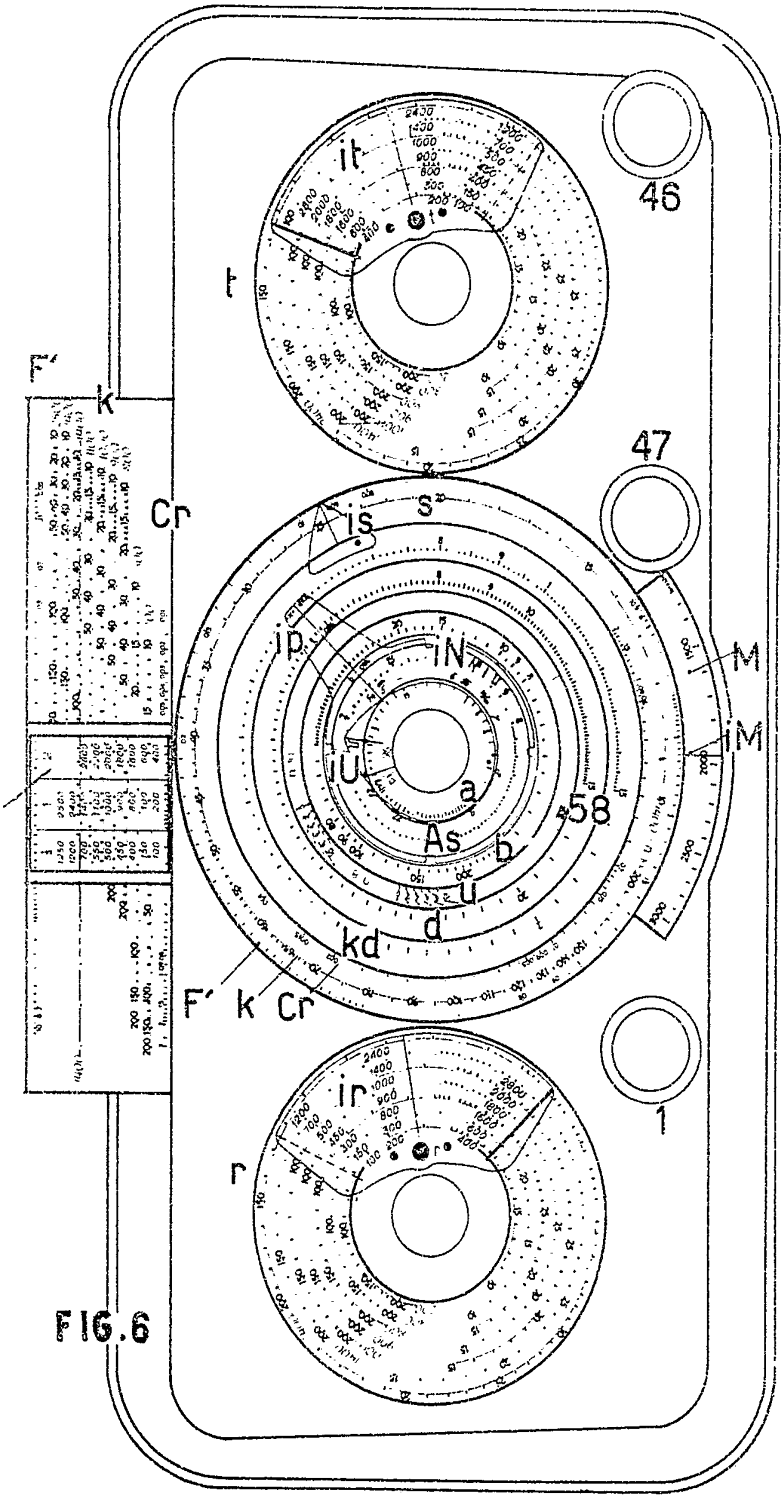
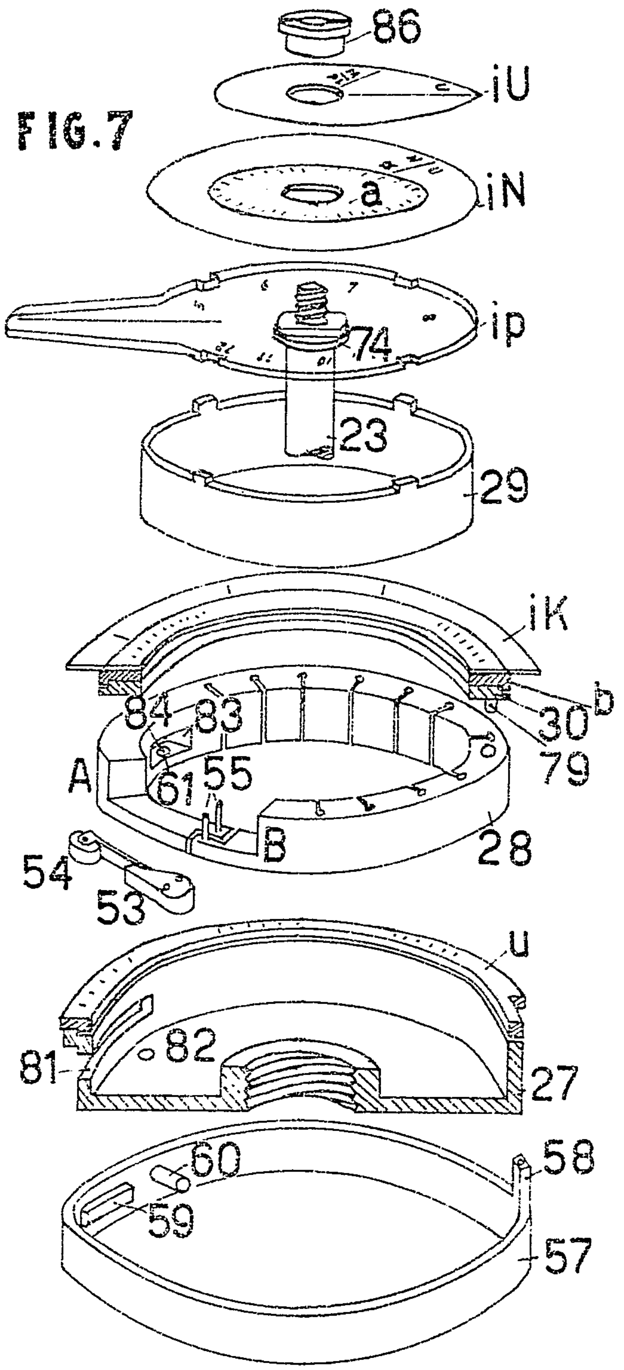


FIG.6

FIG.7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 This drawing is a reproduction of the Original on a reduced scale.
SHEETS 4 & 5

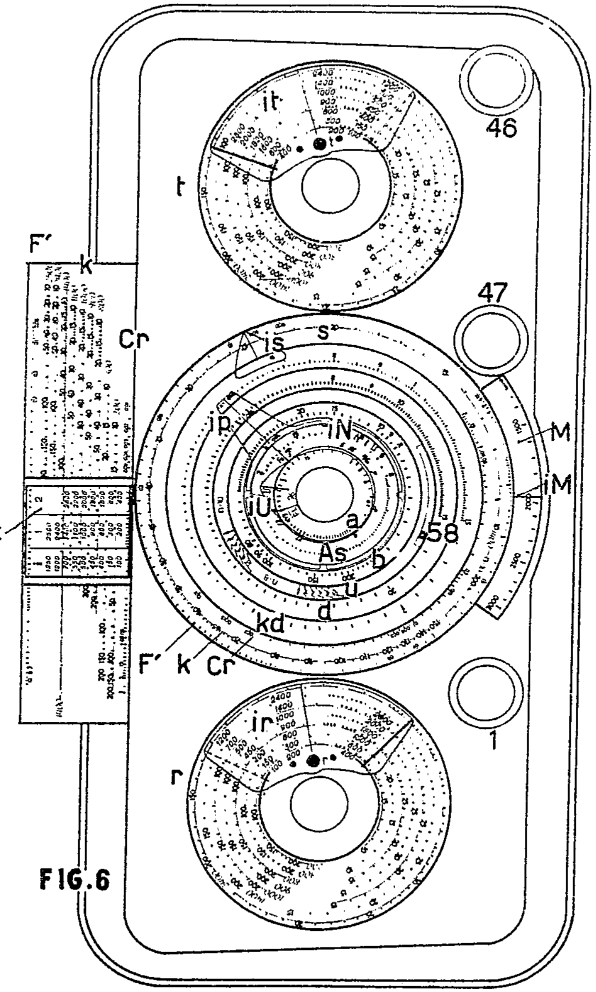

FIG. 6

FIG. 7